**Homework 1**

Question: Consider the rock game for the 16 starting conditions with up to 0,1,2,3 rocks in each pile; determine whether the first player has a winning strategy (i.e., is able to win, no matter what the second player does). Give your reasons, not just the answer.

Answer (Work attached):

First I was able to cut down the number of starting conditions to 10, since the order of the piles didn’t matter. Here are the possible starting conditions with the player that had a winning strategy:

|  |  |  |
| --- | --- | --- |
| Pile 1 | Pile 2 | Winning Strategy |
| 0 | 0 | Tie or N/A |
| 0 | 1 | Player 1 |
| 0 | 2 | Player 2 |
| 0 | 3 | Player 1 |
| 1 | 1 | Player 1 |
| 1 | 2 | Player 1 |
| 1 | 3 | Player 1 |
| 2 | 2 | Player 2 |
| 2 | 3 | Player 1 |
| 3 | 3 | Player 1 |

In order to figure this out, I found that this looked like something I could use some type of proof by induction to answer.

I figured out my base cases, which are the first four configurations (excluding the Tie case). If only one of the piles had rocks in it and it was an odd number, then the winning strategy went to the first player. If, on the other hand, it had an even number the second player would have a winning strategy. Then you had the case where there is one rock in each pile, which would go to the first player since they can remove a rock from each pile.

Every other winning strategy could be determined from the previous configurations. In order to determine if the first player had the winning strategy, there had to be a configuration that the first player can get the rocks into that would leave the second player in a known losing arrangement. If there was no known losing structure the first player could get the second player in, then the second player has the winning strategy. All of my work is found on the next few pages.